Solving Systems with Desmos Homework

Note: you may use a calculator or Desmos for every problem on this homework. No work needs to be shown for either component of part A. However, each component of part B MUST be accompanied by the requested computations/work.

t	0	3	16
Number of Breakfast Tacos	23	40	63

1. The table above gives the number of breakfast tacos sold by a food truck. On the initial day ($t = 0$) when the food truck added breakfast tacos to the menu, the truck sold 23 breakfast tacos. Three days later ($t = 3$), the truck sold 40 breakfast tacos. Thirteen days after that ($t = 16$), the truck sold 63 breakfast tacos.

   The number of breakfast tacos sold can be modeled by the function N given by $N(t) = at^{0.6} + bt + c$, where $N(t)$ is the number of breakfast tacos sold t days after the item was added to the menu.

   A. i. Use the given data to write three equations that can be used to find the values for constants a, b, and c in the expression for $N(t)$.

   ii. Find the values for a, b, and c as decimal approximations.

   B. i. Use the given data to find the average rate of change of the number of breakfast tacos sold, in tacos per day, from $t = 3$ to $t = 16$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

   ii. Use the average rate of change found in part B(i) to estimate the number of breakfast tacos sold on day $t = 20$. Show the work that leads to your answer.

   iii. Explain why using the average rate of change found in part B(i) to approximate $N(t)$ for $t > 16$ leads to an overestimate.

2. A new color has been discovered! It's called shmorple. By three days after its discovery ($t = 3$), 22 million people had already painted their nails shmorple. Twelve days later ($t = 15$), 42 million people had already painted their nails shmorple.

   The number of people who have painted their nails shmorple can be modeled by the function P given by $P(t) = a + b(t + 1)^{\frac{1}{2}}$, where $P(t)$ is the number of shmorple nailed people, in millions, t days after the color was discovered.

   A. i. Use the given data to write two equations that can be used to find the values for the constants a and b in the expression for $P(t)$.

   ii. Find the values for a and b as decimal approximations.

   B. i. Use the given data to find the average rate of change of the number of people who have painted their nails shmorple, in millions of people per day, from $t = 3$ to $t = 15$ days. Express your answer as a decimal approximation. Show the computations that lead to your answer.

   ii. Use the average rate of change found in part B(i) to estimate the number of people who have painted their nails shmorple, in millions, by day $t = 11$. Show the work that leads to your answer.

   iii. Explain why using the average rate of change found in part B(i) leads to an underestimate of $P(t)$ for values of t between $t = 3$ and $t = 15$.

3. Mrs. Hutchinson won the lottery! With her winnings, she begins to collect stones. At first, she is reasonable and only collects a few at a time. But after a while, her thirst for stones grows insatiable and she begins acquiring a massive number of new stones at a time. Two months after winning the lottery ($t = 2$), she owned 49 pounds of stones. Four months later ($t = 6$), she owned 145 pounds of stones. After another 4 months ($t = 10$), Mrs. Hutchinson's stone collection weighed 321 pounds.

   The weight of Mrs. Hutchinson's stone collection can be modelled by the quadratic function S given by $W(t) = at^2 + bt + c$, where $W(t)$ is the weight, in pounds, of Mrs. Hutchinson's collection t months after winning the lottery.

   A. i. Use the given data to write three equations that can be used to find the values for the constants a, b, and c in the expression for $W(t)$.

      ii. Find the values for a, b, and c as decimal approximations.

   B. i. Use the given data to find the average rate of change of the weight of Mrs. Hutchinson's stone collection, in pounds per month, from $t = 2$ to $t = 10$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

      ii. Use the average rate of change found in part B(i) to estimate the weight, in pounds, of Mrs. Hutchinson's stone collection after $t = 12$ months. Show the work that leads to your answer.

4. In March of 2017, the video game console "Nintendo Switch" was released. In its second year of being on the market ($t = 2$), there were 27.5 million sales of Nintendo Switch software in the Americas. In its fourth year of being on the market ($t = 4$), there were 74.3 sales of Nintendo Switch software in the Americas.

   The sales of Nintendo Switch software in the Americas can be modelled by $N(t) = a(t + 1)^2 + bt$ where $N(t)$ is the number of sales, in millions, in year t.

   A. i. Use the given data to write two equations that can be used to find the values of a and b in the expression for $N(t)$.

      ii. Find the values for a and b as decimal approximations.

   B. i. Use the given data to find the average rate of change of the number of Nintendo Switches sold, in millions of consoles per year, from $t = 2$ to $t = 4$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

      ii. Use the average rate of change found in part B(i) to estimate the number of Nintendo Switch units sold in its seventh year on the market. Show the work that leads to your answer.

5. A YouTube celebrity decides he would like to sell energy drinks. In its second month on the market ($t = 2$), the drink made $\$35$ million in sales. In its third month on the market ($t = 3$), the drink made $\$40$ million in sales. In its fifth month on the market ($t = 5$), the drink made $\$26$ million in sales.

   The sales for the drink can be modelled by $S(t) = at^2 + bt + c$ where $S(t)$ is the sales, in millions, made in month t after the product's release.

   A. i. Use the given data to write three equations that can be used to find the values of a, b, and c in the expression for $S(t)$.

      ii. Find the values for a, b, and c as decimal approximations.

   B. i. Use the given data to find the average rate of change of the drink's sales amount, in millions of dollars per month, from $t = 2$ to $t = 5$. Express your answer as a decimal approximation. Show the computations that lead to your answer.

      ii. Use the average rate of change found in part B(i) to estimate how much money the drink made in sales in its 9^{th} month on the market. Show the work that leads to your answer.

Solving Systems with Desmos HW Answers

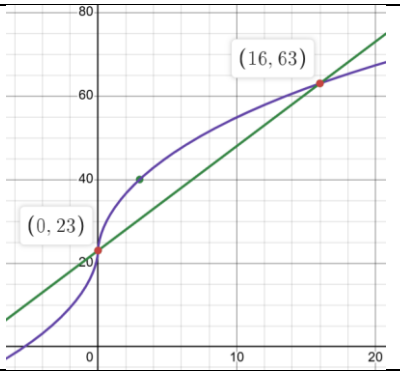
1. A. i.
$$23 = a(0)^{0.6} + b(0) + c$$
$$40 = a(3)^{0.6} + b(3) + c$$
$$63 = a(16)^{0.6} + b(16) + c$$

ii.
$$a = 10.068$$
$$b = -0.821$$
$$c = 23$$

B. i.
$$\frac{N(16) - N(3)}{16 - 3} = \frac{63 - 40}{16 - 3} = \frac{23}{13} = 1.76923$$
$$1.769$$

ii.
$$\frac{N(20) - 63}{20 - 16} \approx 1.76923$$
$$N(20) - 63 \approx 7.0769$$
$$N(20) \approx 70.077$$

iii. For $t > 0$, $N(t)$ appears to be concave down. We are comparing values of the concave down function $N(t)$ to outputs of the secant line passing through N at $t = 0$ and $t = 16$. Since N is concave down, the secant line is above the graph of N after $t = 16$. So, for $t > 16$, the estimates based on the average rate of change overestimate the actual value of $N(t)$.



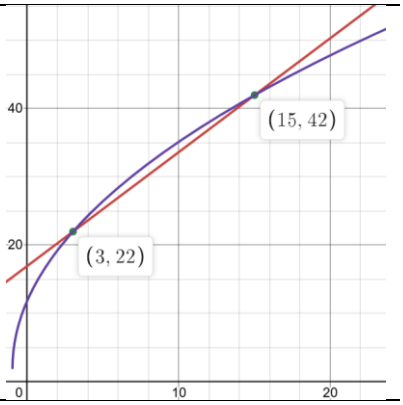
2. A. i.
$$22 = a + b(3 + 1)^{\frac{1}{2}}$$
$$42 = a + b(15 + 1)^{\frac{1}{2}}$$

ii.
$$a = 2$$
$$b = 10$$

B. i.
$$\frac{P(15) - P(3)}{15 - 3} = \frac{42 - 22}{15 - 3} = \frac{20}{12} = \frac{5}{3} = 1.666667$$
$$1.667$$

ii.
$$\frac{P(11) - 22}{11 - 3} \approx \frac{5}{3}$$
$$P(11) - 22 \approx \frac{40}{3}$$
$$P(11) \approx 35.333$$

iii. We are comparing outputs of the concave down function $P(t)$ to outputs of the secant line passing through P at $t = 3$ and $t = 15$. Since P is concave down, the secant line lies below the graph of P between $t = 3$ and $t = 15$. So, estimates based on the average rate of change underapproximate $P(t)$.



3. A. i. $49 = a(2)^2 + b(2) + c$
$145 = a(6)^2 + b(6) + c$
$321 = a(10)^2 + b(10) + c$

ii. $a = 2.5$
$b = 4$
$c = 31$

B. i. $\frac{W(10) - W(2)}{10 - 2} = \frac{321 - 49}{10 - 2} = \frac{272}{8} = 34$

ii. $\frac{W(12) - 321}{12 - 10} \approx 34$
$W(12) - 321 \approx 68$
$W(12) \approx 389$

4. A. i. $27.5 = a(2 + 1)^2 + b(2)$
$74.3 = a(4 + 1)^2 + b(4)$

ii. $a = 2.757$
$b = 1.343$

B. i. $\frac{N(4) - N(2)}{4 - 2} = \frac{74.3 - 27.5}{4 - 2} = \frac{46.8}{2} = 23.4$

ii. $\frac{N(7) - 74.3}{7 - 4} \approx 23.4$
$N(7) - 74.3 \approx 70.2$
$N(7) \approx 144.5$

5. A. i. $35 = a(2)^2 + b(2) + c$
$40 = a(3)^2 + b(3) + c$
$26 = a(5)^2 + b(5) + c$

ii. $a = -4$
$b = 25$
$c = 1$

B. i. $\frac{S(5) - S(2)}{5 - 2} = \frac{26 - 35}{5 - 2} = \frac{-9}{3} = -3$

ii. $\frac{S(9) - 26}{9 - 5} \approx -3$
$S(9) - 26 \approx -12$
$S(9) \approx 14$